Math 250 Review for Final Exam Name______________________

Limits and Continuity

1. a. Sketch the graph of the piecewise function: $f(x)=\begin{cases}-x^2+2, & x>0\\ x-1, & x\le 0\end{cases}$

b. Use the graph to determine the following:

i. $\lim_{x\to 1} f(x)$ *ii.* $\lim_{x\to 0} f(x)$ *iii.* $f(0)$

(2-10) Determine the limits analytically:

2. $\lim_{x\to 3}\dfrac{\sqrt{(x-3)^2}}{x-3}$

3. $\lim_{x\to 0}\dfrac{5x}{x^2-x}$

4. $\lim_{x\to 3}\dfrac{\sqrt{2}-\sqrt{x-1}}{x-3}$

5. $\lim_{x\to 0}\dfrac{\frac{2}{x+3}-\frac{2}{3}}{x}$

6. $\lim_{x\to 0}\dfrac{\sin 3x}{2x}$

7. $\lim_{x\to 2^+}\dfrac{5x}{x-2}$

8. $\lim_{x\to -1^-}\dfrac{x-3}{x+1}$

9. $\lim_{x\to \infty}\dfrac{5x}{x-2}$

10. $\lim_{x\to \infty}\dfrac{3x^2-4x+1}{2-x^3}$

11. Given the graph of $y=f(x)$ shown, determine each of the following.

a. $\lim_{x\to -1^+} f(x)$

b. $\lim_{x\to -1^-} f(x)$

c. $\lim_{x\to 2^+} f(x)$

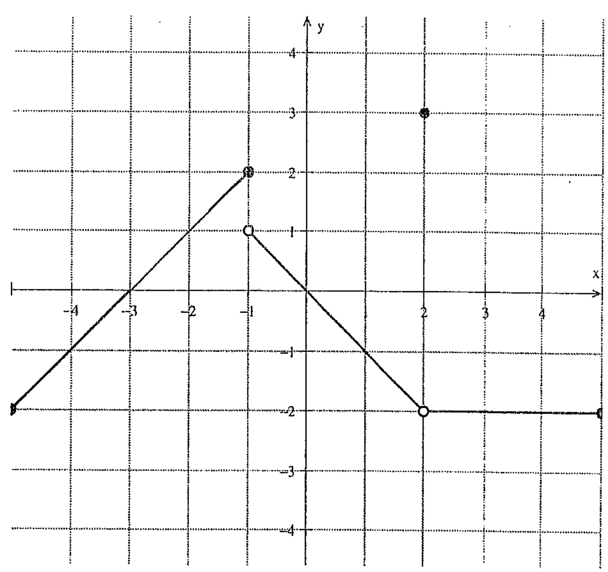
d. $\lim_{x\to 2^-} f(x)$

e. $\lim_{x\to -1} f(x)$

f. $\lim_{x\to 2} f(x)$

g. $f(-1)$

h. $f(2)$



12. Given $f(x)=\begin{cases} 3x-1, & x<-1 \\ 2, & -1\le x<2 \\ x, & x\ge 2 \end{cases}$

a. Identify any POSSIBLE points of discontinuity.

b. Using the FORMAL DEFINITION OF CONTINUITY, justify which of your x-values from part (a) are points of discontinuity, and which are not.

Derivatives!

(1-8) Find each derivative

1. $f(x)=-5x^2+8\sqrt{x}+\dfrac{4}{3x^5}-3$

2. $y=-2\cos x+4\sin x$

3. $g(t)=(4t^2+3)(2t-1)$

4. $P(t)=\dfrac{4}{\sqrt{5-2t}}$

5. $f(x)=x^3\tan x$

6. $M(t)=\dfrac{3t-8}{7-2t}$

7. $P(x)=\dfrac{x}{1-\sin x}$

8. $f(x)=2\csc(3x)+\cot(3x)$

9. **NO CALCULATOR – SHOW WORK!** Write the equation of the line tangent to the curve $y=\sec x$ at $x=\dfrac{\pi}{3}$.

10. Given $P(z)=\csc z$, find $P''(z)$.

11. **NO CALCULATOR.** A particle moves along a vertical line with position function $f(t)=5t^3+\sqrt{t}$, where $f(t)$ is measured in feet, and t is measured in seconds. Find:

a. The velocity at time $t=4$.
b. The acceleration at time $t=4$.

*Be sure to put correct units on your answers to (a) and (b).

12. Given $f(x)=\dfrac{10}{x^3+4}$, find the equation of the line tangent to the function when $x=1$.

13. Given $h(x)=f[g(x)]$, with the values for f and g shown in the table, find the value of $h'(2)$.

	$x=2$	$x=5$
$f(x)$	1	-2
$f'(x)$	-4	9
$g(x)$	5	6
$g'(x)$	-3	-1

14. Given the relation $-8x^2+5xy+y^3=-26$:

a. Find the derivative, $\frac{dy}{dx}$, in simplest form.

b. Find the equation of the line tangent to the curve at the point (1, -2).

15. Given $x^2-3y^2=12$, find $\frac{d^2y}{dx^2}$.

16. The edges of a square are DECREASING at a constant rate of 0.2 in/min. Find the rate of change of the AREA of the square at the moment when the edge is 4 in. long.

17. The radius of a sphere is increasing at a rate of 1.2 cm/sec. Find the rate at which the volume of the sphere is changing at the moment when the radius of the sphere is 6 cm.

18. Sand is falling off a conveyor belt, forming a CONICAL sand pile. The pile forms in such a way that the height of the pile is always 3 times the diameter of the base of the pile. If the sand is falling at a rate of 90 cubic meters per hour, find the rate at which the height of the pile is changing at the moment when the height is 12 meters.

Applications of Derivatives

1. Find the ABSOLUTE MAXIMUM AND MINIMUM VALUES for $f(x)=6x^{2/3}-2x$ on [-1, 1].

2. Consider the function $f(x)=x^2+2x+3$ on the interval [-1, 4].

a. VERIFY that the mean value theorem can be applied to this function on the given interval.

b. Find the value(s) of c guaranteed by the mean value theorem on this interval.

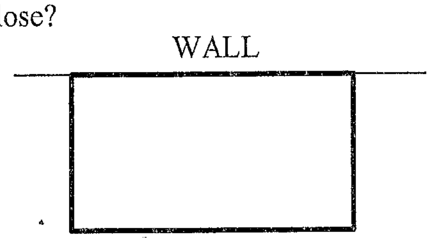
3. PROVE that the function $f(x)=2x^{5/3}-5x^{4/3}$ has exactly one point of inflection, and find that point.

4. Consider the function $f(x) = -x + 2\cos x,\ [0, 2\pi]$

a. Find the intervals on which the function is increasing or decreasing, and the locations (ordered pairs) of any relative extrema.

b. Find the intervals on which the function is concave up or concave down, and the locations (ordered pairs) of any points of inflection.

5. A farmer has 600 feet of fencing with which to enclose a rectangular corral. He is able to use an existing wall for one side, thus only needs to fence three sides. **What is the MAXIMUM AREA** the farmer can enclose?



6. **Find the point** on the curve $y = x^2 - 1$ closest to the point (5, 0).

7. An **open top** box is constructed so that it has a square base. **Find the minimum surface area** given the volume of the box is 12 cubic feet.

8. The radius of a circle is measured to be 5.2 cm, with an error of ± 0.05 cm.

a. Use differentials to approximate the amount of error in the AREA measurement.

b. What is the approximate **percent error** in this AREA measurement?

9. a. Find the linear approximation to $f(x) = \sqrt{x}$ at $x = 25$.

b. Use your linear approximation to approximate the value of $\sqrt{25.5}$. Show your work – no decimals (use fractions).

Integration:

Evaluate each integral.

1. $\int (x^3 + 4x - 5)dx$

2. $\int \sec x \tan x dx$

3. $\int \cos(4x)dx$

4. $\int \frac{x}{(x^2 - 5)^4} dx$

5. $\int \tan^3 x \sec^2 x dx$.

6. $\int_0^3 (1-x)dx$

7. $\int_{-1}^{4} |x-1|\, dx$

8. $\int_0^{\pi/3} (2\sin x + \cos x)dx$

9. $\int_0^8 \sqrt{3x+1}dx$

10. Find the average value of the function $f(x) = \frac{4}{\sqrt{x}}$ on the interval [1, 9]

Logarithmic and Exponential Functions

(1-2) Integrate.

1. $\int \frac{1}{3-2x} dx$

2. $\int \tan(5x)\, dx$

3. Evaluate the definite integral. Show your work - no calculator – give your answer in exact form.

$$\int_0^4 \frac{8x}{x^2+1} dx$$

(4-5) Differentiate.

4. $y = 4e^{-3x}$

5. $f(x) = e^{2x} \ln x$

(6-7) Integrate.

6. $\int 4e^{-3x} dx$

7. $\int \frac{e^x}{(1+e^x)^3} dx$

8. Find dy/dx: $e^{2y} + e^{2x} = 4$

Area and Volume

(1-4) Consider the region enclosed by $y = x^2 + 1$ and $y = 3x + 5$.

1. Sketch a picture to show the region, and **find the points of intersection by hand.**

2. Set up an integral that can be used to find the AREA of this region. **You do not need to evaluate your integral, but should simplify the integrand.**

3. Set up an integral that can be used to find the VOLUME of the solid formed if this region is revolved about the x – axis. **You do not need to evaluate the integral, and you do not need to simplify the integrand.**

4. Set up an integral that can be used to find the VOLUME of the solid formed if this region is revolved about the line $y = -2$. **You do not need to evaluate the integral, and you do not need to simplify the integrand.**

5. Find area of the region enclosed by $y = \sqrt{x}$, $y = 1$, $y = 2$, and the y axis. Draw and shade the region. **Evaluate your integral BY HAND.**