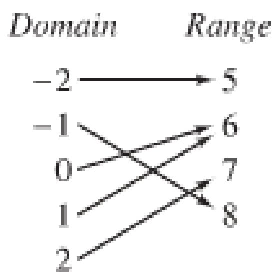
Math 121 Practice Final Exam

***Disclaimer: The actual Math 121 final exam for your class may include, but is not limited to, the problems provided in this handout. Please be sure to study your notes, homework assignments, quizzes, and tests to fully prepare for your final. Also, see a tutor and/or your professor if you need any additional assistance.**

1. Southwestern College invests \$500,000 a savings account at 3.5%, compounded semiannually. How much is in the account at the end of 15 years?

   \$841,400.07 is in the account after 15 years.

2. Determine whether or not each correspondence is a function. If not, explain why.

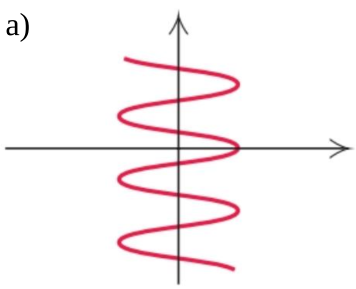
   a) 

   Function

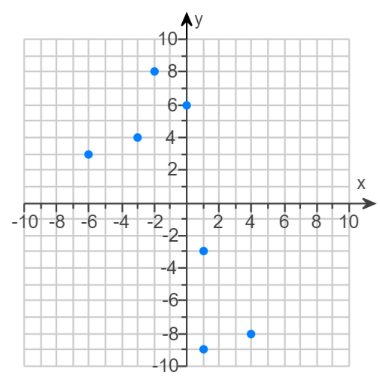
   b) Domain: students in a class
   Correspondence: grade received on an exam
   Range: $\{A, B, C, D, F\}$

   Function

3. Determine whether each of the following is a graph of a function. If not, explain why.

   a) 

   No, fails the V.L.T.

   b) 

   No, fails the V.L.T.

4. Write each interval or set in the specified <u>notation</u> and <u>graph it</u> on the real number line.

   a) $(-3, 5]$

   Set Notation: $\{x \mid -3 < x \leq 5\}$

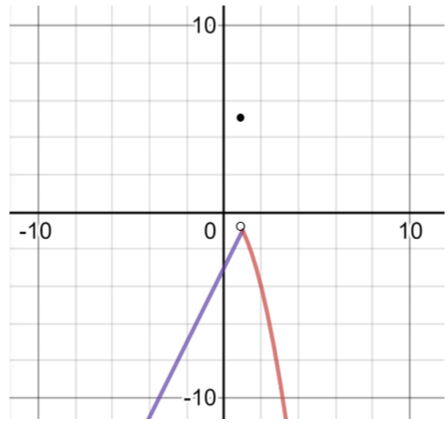
   Number Line: 

   b) $\{x \mid -4 < x < 5\}$

   Interval Notation: $(-4, 5)$

   Number Line: 

5. Graph the function $f(x)=\begin{cases}2x-3, & \text{for } x<1,\\ 5, & \text{for } x=1,\\ -x^2, & \text{for } x>1.\end{cases}$



6. Given the function $f(x)=3-x-3x^2$, find the following values.

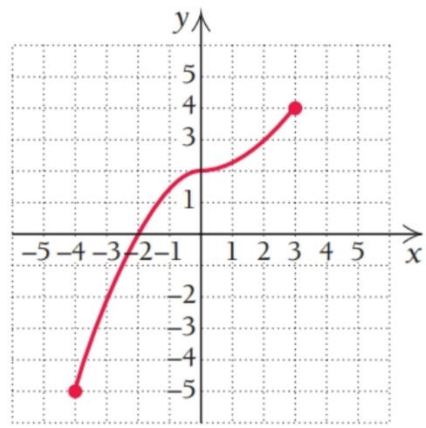
a) $f(x+h)$

$3-x-h-3x^2-6xh-3h^2$

b) $\dfrac{f(x+h)-f(x)}{h}$

$-1-6x-3h$

7. For the graph of the function *f*, do the following:



a) Find $f(-3)$. -2

b) The domain of *f*. $[-4,3]$

c) The range of *f*. $[-5,4]$

8. Find the domain of the following functions.

a) $f(x)=3x^2+2$

$(-\infty,\infty)$

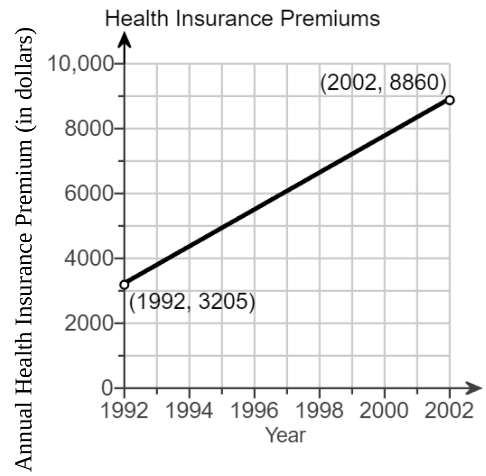
b) $f(x)=\dfrac{3-x}{2-7x}$

$\left\{x \mid x\neq \dfrac{2}{7}\right\}$

9. Find the domain of $h(x)=\sqrt{-2x+9}+\sqrt{x+2}$. $\left[-2,\dfrac{9}{2}\right]$

10. Find the average rate of change in the annual premium for a family's health insurance.

The average rate of change in the annual premium is $565.50 per year.



11. Online store charges \$20 to ship orders for which the total amount spent is any amount up to and including \$100. For every extra \$20 spent, the shipping rate is reduced by \$5, with the possibility of free shipping if the total amount of the order is large enough. Let S(x) be the shipping charge, in dollars, for an order totaling x dollars.

a) Find S(55), and explain what that value represents.

The shipping cost for an order totaling \$55 is \$20.

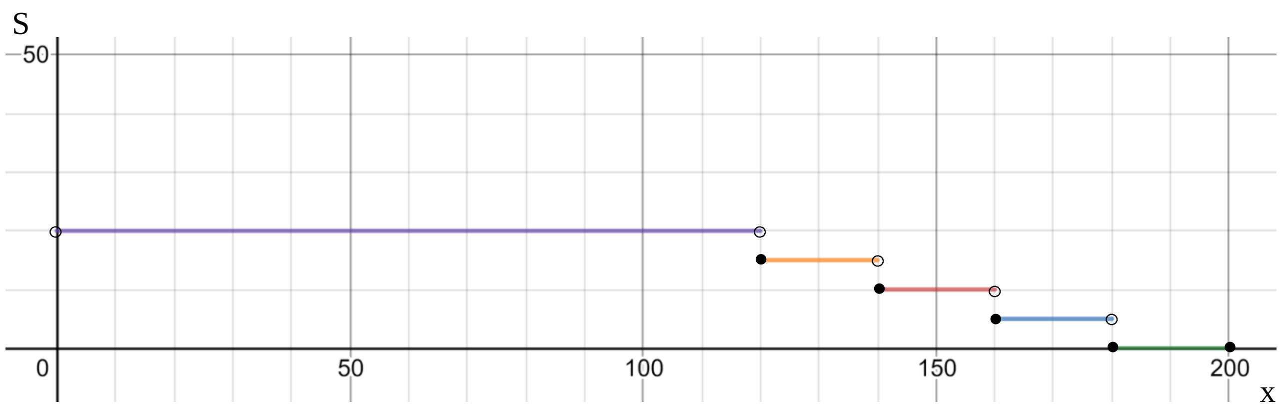
b) Find S(120), and explain what that value represents.

The shipping cost for an order totaling \$120 is \$15.

c) What is the minimum order total to receive free shipping?

The shipping cost for an order totaling \$180 is free.

d) Sketch the graph of S for $0 < x \leq 200$.



12. A registrar's office finds that the number of inkjet cartridges, I, required each year for its copiers and printers is **directly proportional** with the number of students enrolled, s.

a) Find an equation that expresses I as a function of s, if the office requires 19 cartridges when 2600 students enroll. Round to 4 decimal places.

$I = 0.0073s$

b) How many cartridges would be required if 3100 students enrolled? Round to the nearest whole number.

23 cartridges would be needed.

Points: /10

13. Find the equation of a line given the following conditions. Write your answer in Slope-Intercept Form:

a) goes through the points $(-3,8)$ and $(-7,-1)$.

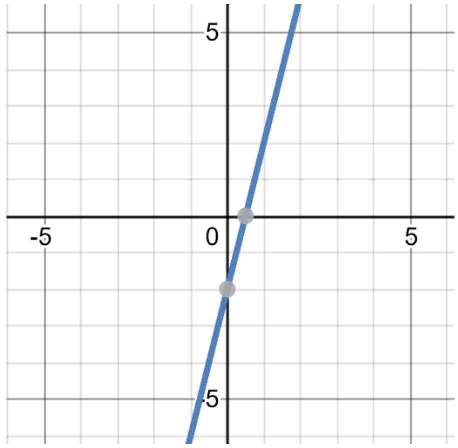
$y = \frac{9}{4}x + \frac{59}{4}$

b) m = 0 and goes through the point $(7,-10)$.

$y = -10$

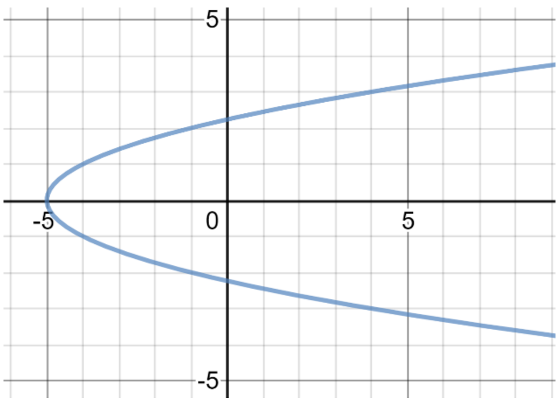
14. Find the slope, y-intercept, and then graph the equation $20x - 5y - 10 = 0$.

m = 4   y-int: $(0, -2)$

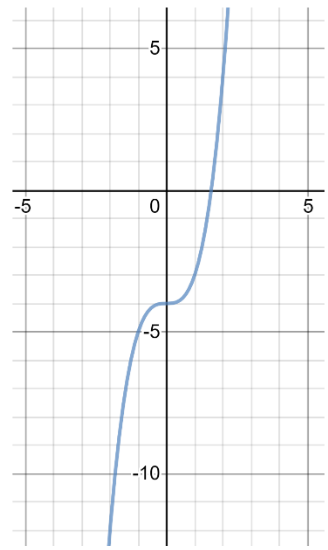


15. Graph the following:

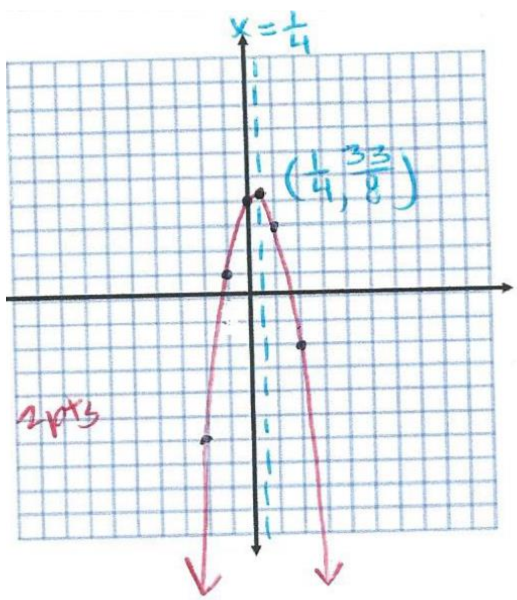
a) $x = y^2 - 5$



b) $y + 4 = x^3$

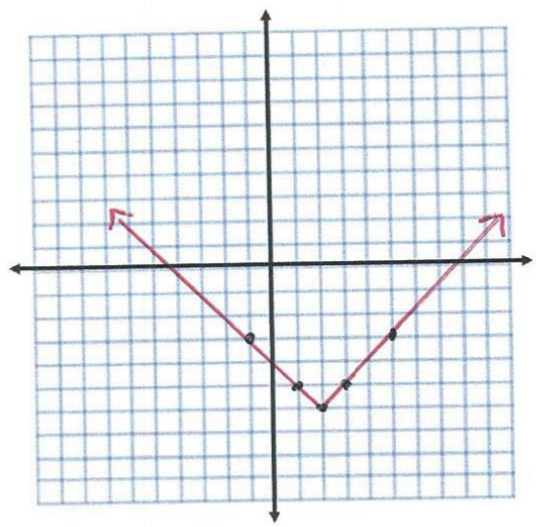


16. Graph $g(x) = -2x^2 + x + 4$, and determine and label the vertex and line of symmetry.

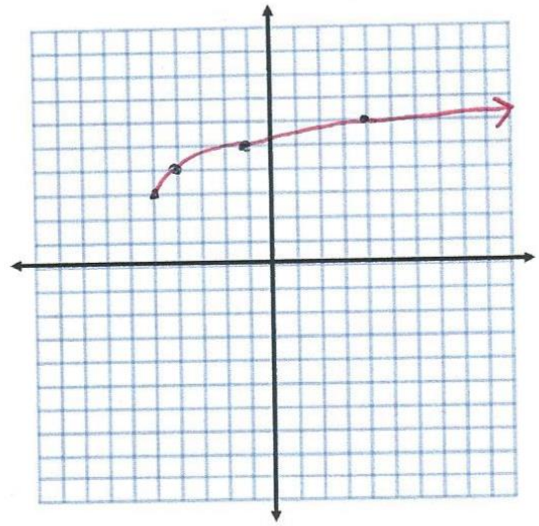


17. Graph the following:

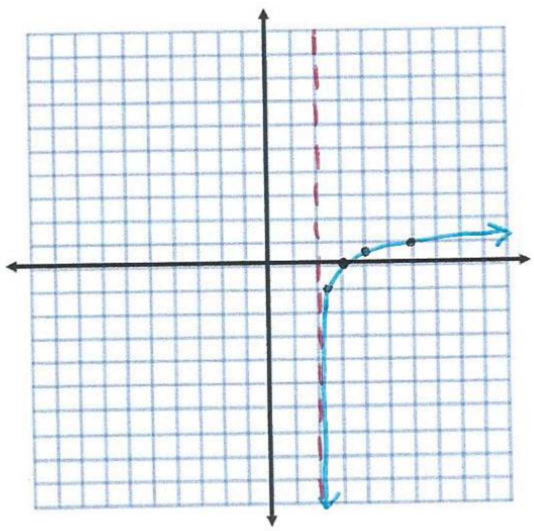
a) $f(x) = |x-2| - 6$



b) $f(x) = \sqrt{x+5} + 3$



18. Graph $y = \log_4(x-2)$, and state its domain.



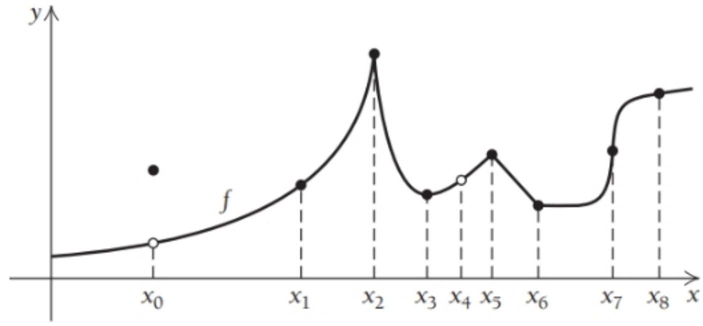
Domain: $(2, \infty)$

19. Use the quadratic formula to solve $4x^2 = 4x + 1$. $x = \dfrac{1 \pm \sqrt{2}}{2}$

20. The quantity sold q of a high-definition TV is inversely proportional to the price p. If 85,000 high-definition TVs sold for $1200 each, how many will be sold if the price is $850 each

120,000 TVs are sold when the price is $850.

21. For the graph, list all the x-values for which the function is not differentiable.



$x_0, x_2, x_4, x_5, x_6, x_7$

22. Find the equilibrium point for the demand and supply functions. $Demand: p = 1000 - 10q$
$Supply: \ p = 250 + 5q$

$(50, 500)$

23. The startup GorePoint Inc. had 15 employees initially and 50 employees 6 months later. Assume that the number of employees increases by the same percentage per month.

a) Find the exponential function G that gives the number of employees t months after the company started operations. Round to 2 decimal places, if needed.
$G(t) = 15(1.22)^t$

b) When will the number of employees at GorePoint Inc. reach 250? Round to the nearest month.
In 14 months

24. Given $\log_a 2 = 0.483$ *and* $\log_a 3 = 0.766$, find each of the following:

a) $\log_a 18 =$ 2.015

b) $\log_a \frac{1}{3}$ = -0.766

25. Solve for x. Round answer to 4 decimal places, if needed.

a) $5^x = 50$
x = 2.4307

b) $\log_x 125 = 3$
x = 5

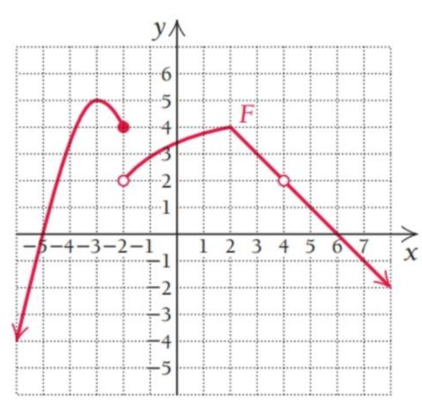
26. Find the limit.

a) $\lim_{x \to 5} \dfrac{\frac{1}{x} - \frac{1}{5}}{x - 5}$ $\quad \dfrac{-1}{25}$

b) $\lim_{h \to 0} \dfrac{5x^4h - 9xh^2}{h}$ $\quad 5x^4$

27. Determine the value for c so that $\lim_{x \to 3} f(x)$ exist. $f(x)\begin{cases} \frac{1}{3}x + c, & for\ x < 3 \\ -x + 10, & for\ x > 3 \end{cases}$ c = 6

28. Use the graph of F to find each limit.



a) Find $\lim_{x \to -2^+} F(x)$ = 2

b) Find $\lim_{x \to -2} f(x)$ DNE

c) Find $\lim_{x \to 4} f(x)$ = 2

29. Find the limit for the following:

a) $\lim_{x\to -1}\left(3x^5+4x^4-3x+6\right)$ = 10

b) $\lim_{x\to -3}\frac{2x^2-x-21}{x^2-9}$ = 13/6

c) $\lim_{x\to -\infty}\left(\frac{4x^5+x^4+2x-3}{28x^5+x^4-5}\right)$ = 1/7

d) $\lim_{h\to 0}\left(\frac{2}{4x^2+xh-h^2}\right)=\frac{1}{2x^2}$

30. Use the 3 Rules of Continuity to show whether each function given is continuous at the value $x=5$.

a) $f(x)=\begin{cases}\frac{x^2-4x-5}{x-5}, & for\ x<5,\\ x+1, & for\ x\geq 5.\end{cases}$

1) $f(5)=6$, $f(5)$ *exist*

2) $\lim_{x\to 5^-} f(x)=6$

$\lim_{x\to 5^+} f(x)=6$

$\lim_{x\to 5} f(x)$ *exist*

3) $\lim_{x\to 5} f(x)=f(5)$

f *is continuous at* $x=5$

b) $f(x)=3x-2$

1) $f(5)=13$, $f(5)$ *exist*

2) $\lim_{x\to 5} f(x)=13$

$\lim_{x\to 5} f(x)$ *exist*

3) $\lim_{x\to 5} f(x)=f(5)$

f *is continuous at* $x=5$

31. For $f(x)=\frac{2}{x}$, find the equation for the tangent line at x = 1. $y=-x+3$

32. Determine the equation of the tangent line to the function $f(x)=e^{x^2-1}$ at x = 1. $y=2x-1$

33. Find the equation of the tangent line to the curve $y^2=x^3-4x+1$ at the point $(-2,1)$ using implicit differentiation. $y=4x+9$

34. Find the derivative of the following functions:

a) $f(x)=\left(5x-\sqrt[4]{x}+\frac{3}{x}-4\right)$ $\quad 5-\frac{1}{4}x^{-3/4}-3x^{-2}$

b) $f(x)=\frac{x^2-16}{x^2+4}$ $\quad \frac{40x}{(x+4)^2}$

c) $g(x)=\left(x^5-3x^4-2\right)\left(x^3-7x\right)$

$8x^7-21x^6-42x^5+105x^4-6x^2+14$

d) $y=\frac{1}{\left(2-5x^2\right)^3}$ $\quad 30x\left(2-5x^2\right)^{-4}$

e) $h(x)=-2\left(x^2-4x+9\right)^6$

$-12(2x-4)\left(x^2-4x+9\right)$

f) $f(x)=\sqrt{x^2+\sqrt{1-3x}}$

$\frac{1}{2}\left(x^2+(1-3x)^{1/2}\right)^{-1/2}\left(2x-\frac{3}{2}(1-3x)^{-1/2}\right)$

35. Find the derivative of $f(x)=\sqrt[5]{\frac{3+2x}{5-x}}$

$\frac{13(3+2x)^{-4/5}}{5(5-x)^{6/5}}$

36. Find d^2y/dx^2 for $y=(x^3-2)^5$. $30x(x^3-2)^3(7x^3-2)$

37. Find $\frac{d^4y}{dx^4}\left[\frac{1}{x}\right]=24x^{-5}$.

38. Find any points on the graph of $f(x)=-x^3+6x^2$ at which the tangent has a slope of 9. $(1,5),(3,27)$

39. The graph shows the distance of Jesse's car from home as a function of time t.

Find the interval(s) in which Jesse is:

a) traveling at a constant velocity;

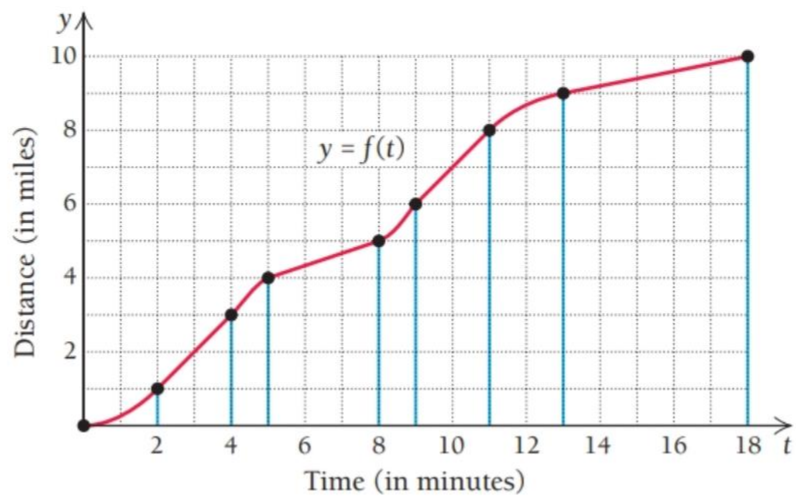
$(2,4),(5,8),(9,11),(13,18)$

b) accelerating (speeding up);

$(0,2),(8,9)$

c) decelerating (slowing down)

$(4,5),(11,13)$



40. From the point at which an object is dropped, the distance it falls in t seconds, assuming negligible air resistance, is approximately $s(t)=16t^2$, where s(t) is in feet. If a stone is dropped from a cliff, find each of the following, assuming that air resistance is negligible:

(a) how far the stone has traveled 4 sec after being dropped, 256ft

(b) how fast it is traveling 4 sec after being dropped, and 128ft/s

(c) its acceleration after it has been falling for 4 sec. 32ft/second squared

41. Future Value. Luis invests $1500 in an account that earns interest at an annual rate of 3.25%. Find the future value of Luis's account after 5 yr if interest is compounded:

a) quarterly

$1763.51

b) continuously

$1764.67

42. Given $\ln 4=1.3863$ *and* $\ln 5=1.6094$, Use properties of natural logarithms to find each of the following:

a) $\ln\frac{1}{4}$

-1.3863

b) $\ln 80$

4.382

c) $\ln\sqrt{e^8}$

4

43. Find the domain of $g(x)=\ln(x+6)+\ln(x)$.

$(0,\infty)$

44. Solve the logarithmic equation: $\ln(x^2-25)-\ln(x+5)=\ln 1$ $\{6\}$

45. Revenue from sales of digital content by the New York Times doubled over the 6 year period between 2011 and 2017. Assuming growth of the digital revenue during this period is exponential, what was the exponential growth rate?

11.6%

46. Differentiate each of the following with respect to x:

a) $y = e^{\sqrt{2x^4-3}}$

$y' = 4x^3\left(2x^4-3\right)^{-1/2} e^{\left(2x^4-3\right)^{1/2}}$

b) $s(t) = \dfrac{t^2}{e^{3t}}$

$s'(t) = \dfrac{t(2-3t)}{e^{3t}}$

c) $f(x) = x^7 e^{4x}$

$f'(x) = x^6 e^{4x}(7+4x)$

d) $y = x^6 \ln x - x$

$y' = 6x^5 \ln x + x^5 - 1$

e) $y = \ln\left(7x^2+5x+2\right)$

$y' = \dfrac{14x+5}{7x^2+5x+2}$

f) $y = \ln\left(\dfrac{x^2+5}{x}\right)$

$y' = \dfrac{2x}{x^2+5} - \dfrac{1}{x}$ *or* $y' = \dfrac{x^2-5}{x\left(x^2+5\right)}$

g) $f(x) = \ln\sqrt[5]{x^3-1}$ $\qquad \dfrac{3x^2}{5\left(x^3-1\right)}$

47. Suppose P_0 , in dollars, is invested in a Bond Fund, with interest compounded continuously at 5.9% per year. That is, at any point in time after t years, the balance P, in dollars per year, is growing at the rate $\dfrac{dP}{dt} = 0.059P.$

a) Find the function that satisfies the equation. Write it in terms of P_0 and 0.059.

$P(t) = P_0 e^{0.059t}$

b) Suppose that \$1000 is invested. What is the balance after 1 yr?

After 1 year, there is \$1060.78 in the account.

c) If \$1000 is invested, how fast is the balance growing at t = 2 yr?

The balance is increasing \$66.39 per year.

48. A bottle of soda with a temperature of 34°F is placed in a room in which the temperature is 75°F and allowed to warm naturally. After 15 minutes, the soda's temperature is 45°F. Use $P(t) = L + Pe^{-kt}$.

a) Find P(t), the soda's temperature after t minutes; Round to 4 decimal places if needed.

$P(t) = 75 - 41e^{-0.0208t}$

b) Find P'(20) and interpret its meaning; Round to 2 decimal places.

After 20mins, the temperature is increasing 0.56°F per min.

c) Find the time at which the soda's temperature will reach 70°F. Round to 1 decimal place.

The soda will reach 70°F in 101.2 mins.

49. Forensics: Determining Time of Death. A body is found slumped over a desk in a study. The coroner arrives at noon, immediately takes the temperature of the body, and finds it to be 94.6°. She waits 1 hr, takes the temperature again, and finds it to be 93.4°. She also notes that the temperature of the room is 70°. What was the approximate time of death if the bodies normal body temperature is 98.6°? Use $T(t) = ae^{-kt} + C$. Round your final answer to the nearest hour.

3 hours earlier or 9am.

50. Life Science: Spread of an Epidemic. In a town with a population of 3500, an epidemic of a disease occurs. The number of people, N, infected t days after the disease first appears is given by

$$N(t) = \frac{3500}{1 + 19.9e^{-0.6t}}.$$

a) How many people are initially infected with the disease (t=0)? 167 infected people.

b) Find the number of residents infected after 18 days. 3499 infected people after 18 days.

c) Find the rate at which the disease is spreading after 16 days. 2.8 people / day

d) Will all 3500 residents ever be infected? Why or why not? Yes, taking the limit of the function as time goes to infinity will result in the entire population of 3500 people being infected.

51. Find any relative extrema for the function $f(x) = x^3 + 6x^2 - 36x - 60$ and sketch the graph.

a. Find any critical points. $(2, -100)$ *and* $(-6, 156)$

b. Use the 1st Derivative Test to identify any relative max/mins.

$f'(-7) = 27$

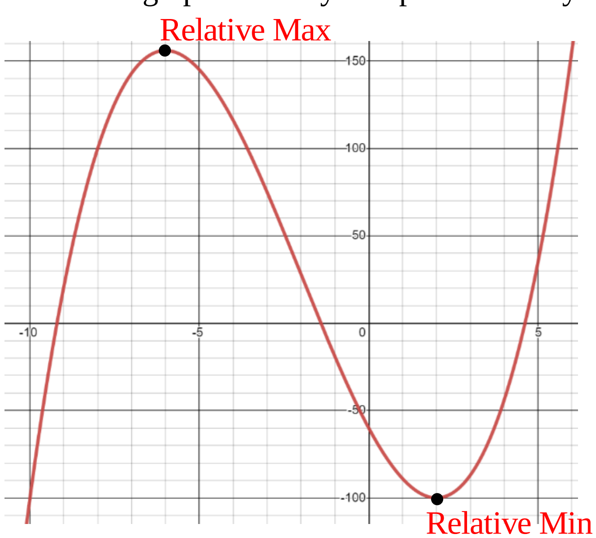
$f'(0) = -36$

$f'(3) = 27$

Relative Max is 156 at $x = -6$

Relative Min is -100 at $x = 2$

c. Sketch the graph. Label your spaces and any rel. max/min points.



52. Find any relative extrema and inflection points for the function $f(x)=e^{-x^2}$ and sketch the graph.

a. Find any vertical, horizontal, and slant asymptotes. V.A. None, H.A. y = 0, S.A. None

b. Find any critical points. $(0,1)$

c. Use the 1st Derivative Test to identify any relative max/mins.

$f'(-1)=0.7358$, f is $\uparrow$ on $(-\infty,0)$

$f'(1)=-0.7358$, f is $\downarrow$ on $(0,\infty)$

Relative maximum is 1 at x=0.

d. Find any possible inflection points. $(-0.707, 0.607)$ and $(0.707, 0.607)$

e. Use the Concavity Test to identify any inflection points.

$f''(-1)=0.736$, f is concave $\uparrow$ on $\left(-\infty,-\sqrt{\frac{1}{2}}\right)$

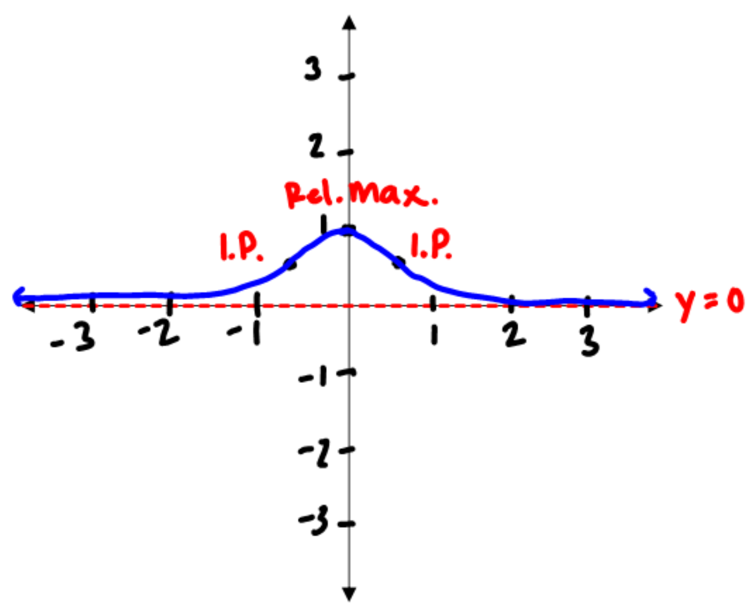
$f''(1)=0.736$, f is concave $\uparrow$ on $\left(\sqrt{\frac{1}{2}},\infty\right)$

$f''(0)=-2$, f is concave $\downarrow$ on $\left(-\sqrt{\frac{1}{2}},\sqrt{\frac{1}{2}}\right)$

$(-0.707, 0.607), (0.707, 0.607)$

Both are inflection pts.

f. Sketch the graph. Label your spaces, inflection points, and any rel. max/min points.



53. Find any relative extrema and inflection points for the function $f(x) = \ln(x^2 + 6)$ and sketch the graph.

a. Find any critical points. $(0, 1.79)$

b. Use the 1st Derivative Test to identify any relative max/mins.

$f'(-1) = -\frac{2}{7}$

$f'(1) = \frac{2}{7}$ Relative min at $(0, 1.79)$

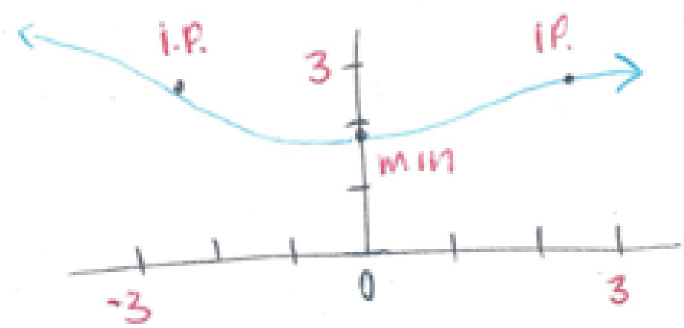
c. Find any possible inflection points. $(2.45, 2.48)$ *and* $(-2.45, 2.48)$

d. Use the Concavity Test to identify any inflection points.

$f''(-3) = -\frac{2}{75}$, $f''(0) = \frac{1}{3}$, $f''(3) = -\frac{2}{75}$

Inflection points at $(2.45, 2.48)$ and $(-2.45, 2.48)$

e. Sketch the graph. Label your spaces, inflection points, and any rel. max/min points.



54. Find any relative extrema for the function $f(x) = \dfrac{-2x^2}{x^2 - 25}$ and sketch the graph.

a. Find any vertical, horizontal, and slant asymptotes.
$V.A.\ x = -5\ and\ x = 5,\quad H.A.\ y = -2,\ S.A.\ None$

b. Find any critical points. $(0,0)$

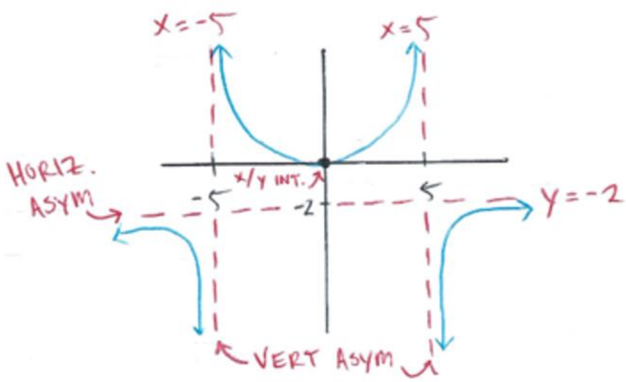
c. Use the 1st Derivative Test to identify any relative max/mins.

$f'(-1) = -\dfrac{25}{144}$

$f'(1) = \dfrac{25}{144}$ Relative Min at $(0,0)$.

d. Find any intercepts. Both x and y-intercept at $(0,0)$.

e. Sketch the graph. Label your spaces and any rel. max/min points.



55. Find any vertical or slant asymptotes.

a) $f(x) = \dfrac{x^2 - 4}{x - 1}$ $S.A.\ y = x + 1$

b) $f(x) = \dfrac{2x^2 + x - 1}{x - 3}$ $S.A.\ y = 2x + 7$

56. Find the absolute extreme values for the function $f(x) = e^{3x} - e^{2x} on\ [-2,1]$.

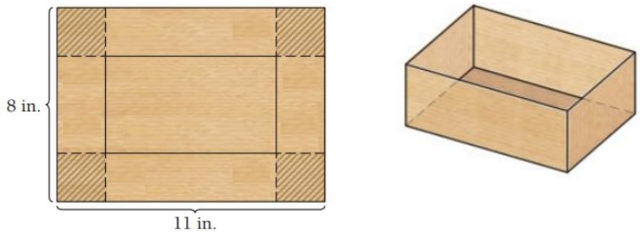
$Abs.\ min\ is\ -0.148\ at\ x = ln\dfrac{2}{3}.$

$Abs.\ max\ is\ 12.696\ at\ x = 1.$

57. A lifeguard has 2500 meters of roped-together flotation devices with which to enclose a rectangular swimming area at the beach. If the shoreline forms one side of the rectangle, what dimensions will the maximize the size of the area for swimming? Also, state the maximum area.

625 meters by 1250 meters will result in a maximum area of 781,250 sq. meters.

58. From a sheet of cardboard, 8 inches by 11 inches, squares are cut out at the corners so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume?



The dimensions of the box that maximizes the volume is 7.95in by 4.95in by 1.525in or cut out a square with 1.525in sides from each corner and fold up the flaps .

The maximum volume is 60.0125 in^3.

59. Find dy/dx by implicit differentiation for each function:

a) $x^3 - xy = y^3$ $\quad \dfrac{dy}{dx} = \dfrac{3x^2 - y}{x + 3y^2}$

b) $\dfrac{2x + y}{x - 5y} = 1$ $\quad \dfrac{dy}{dx} = \dfrac{-1}{6}$

60. For the demand equation $x = \sqrt{2500 - p^2}$ where *x* is number of units sold and *p* is the price in dollars, use implicit differentiation to find *dp/dx*. Then evaluate *dp/dx* at p = 40 and interpret your answer.

An increase in the price by \$0.75 will increase sales by 1 unit.

61. The variables x and y are differentiable functions of t and are related by the equation $3y = 2x(x - 10)$. Find dx/dt ,when x = 2, and dy/dt = 30.

$\dfrac{dx}{dt} = -7.5$

62. A cube of ice is melting so that each edge is decreasing at the rate of 2 inches per hour. Find how fast the volume of the ice is decreasing at the moment when each edge is 10 inches long.

$V = x^3$ When the edge is 10 inches long, the volume is decreasing 600 cubic inches per hour.

63. A snowball is melting so that its radius is decreasing at a rate of 4 inches per hour. How fast is the volume decreasing when the radius is 3 inches? Use the formula for volume of a sphere with radius r: $V = \dfrac{4}{3}\pi r^3$

The volume is decreasing 144π in^3/hr when the radius is 3 inches.

64. If the radius of a spherical balloon increases at a rate of 1.5 inches per minute, find the rate at which the surface area changes when the radius is 6 inches.

Formula for surface area: $S = 4\pi r^2$

The surface area is increasing 72π in^2/min when the radius is 6 inches.

65. The radius of a right circular cylinder is increasing at the rate of 2 inches per second, while the height is decreasing at the rate of 10 inches per second. At what rate is the volume of the cylinder changing when the radius is 16 in. and the height is 6 in.?

$V = \pi r^2 h$ $\quad -2176\pi\ \text{cubic inches / second}$