MATH 120 Final Study Guide Solution Key

PSP -Ana-

1. $\int (4x^{11} - 7x^3 + 8)\,dx$

$= \int 4x^{11}dx - \int 7x^3 dx + \int 8\,dx$ — You can separate each integral (Sum/Difference)

$= 4\int x^{11}dx - 7\int x^3 dx + 8\int dx$ — You can also take out the constant.

$= \frac{\cancel{4}^{1}}{1} \cdot \frac{x^{12}}{\cancel{12}_{3}} - 7 \cdot \frac{x^4}{4} + 8 \cdot x + C$ — Simplify.

$= \boxed{\frac{1}{3}x^{12} - \frac{7}{4}x^4 + 8x + C}$ C

2. $\int \frac{39}{x^2}dx$

$= 39\int \frac{1}{x^2}dx$

$= 39\int x^{-2}dx$

$= \frac{39}{1} \cdot \frac{x^{-1}}{-1} + C$

$= -39x^{-1} + C$

$= \boxed{-\frac{39}{x} + C}$ C

3. $f'(x) = x - 2$, $f(1) = 11$

$f(x) = \int f'(x)\,dx$

$= \int (x-2)\,dx$

$= \int x\,dx - 2\int dx$

$f(x) = \frac{x^2}{2} - 2x + C$

$\boxed{f(x) = \frac{x^2}{2} - 2x + \frac{25}{2}}$ D

To satisfy initial condition

Plug in $f(1) = 11$,

$f(1) = \frac{(1)^2}{2} - 2(1) + C$

$11 = \frac{1}{2} - \underbrace{\frac{2}{1}}_{\text{Subtract}} + C$

$11 = \frac{1}{2} - \frac{4}{2} + C$

$11 = -\frac{3}{2} + C$

$+\frac{3}{2} \quad +\frac{3}{2}$

$\frac{22}{2} + \frac{3}{2} = C$

$C = \frac{25}{2}$

Plug in

4. $f'(x) = 5x^2 - 7x + 4$, $f(0) = 2$

$f(x) = \int f'(x)\,dx$

$= \int (5x^2 - 7x + 4)\,dx$

$= 5\int x^2 dx - 7\int x\,dx + 4\int dx$

$= \frac{5}{1} \cdot \frac{x^3}{3} - \frac{7}{1} \cdot \frac{x^2}{2} + 4x + C$

$f(x) = \frac{5}{3}x^3 - \frac{7}{2}x^2 + 4x + C$

$\boxed{f(x) = \frac{5}{3}x^3 - \frac{7}{2}x^2 + 4x + 2}$ B

$f(0) = \frac{5}{3}(0)^3 - \frac{7}{2}(0)^2 + 4(0) + C$

$f(0) = C$

$C = 2$

Plug in

5. To find $C(x)$, given marginal cost $C'(x) = 10x - 4$ and fixed cost $C(0) = 2$,

*Fixed cost means cost is \$2, even when $x=0$.

$C(x) = \int C'(x)\,dx$

$= \int (10x - 4)\,dx$

$= 10\int x\,dx - 4\int dx$

$= \frac{10}{1} \cdot \frac{x^2}{2} - 4x + C$

$C(x) = \frac{10}{2}x^2 - 4x + C$

$C(x) = 5x^2 - 4x + C$

$\boxed{C(x) = 5x^2 - 4x + 2}$ C

To find C

$C(0) = 5(0)^2 - 4(0) + C$

$C(0) = 2$

$2 = 0 - 0 + C$

$C = 2$

6. Given $R'(x) = 4x^2 - 4$, with initial revenue of 0 $[R(0) = 0]$, find $R(x)$.

$$R(x) = \int R'(x)\,dx$$
$$= \int (4x^2 - 4)\,dx$$
$$= 4\int x^2\,dx - 4\int dx$$
$$= \frac{4}{1}\cdot\frac{x^3}{3} - 4x + C$$
$$= \frac{4}{3}x^3 - 4x + C$$

$$\boxed{R(x) = \frac{4}{3}x^3 - 4x}$$ D

Find C

$$R(0) = \frac{4}{3}(0)^3 - 4(0) + C$$
$$R(0) = C$$
$$C = 0$$

Plug in

7. $\int (3+t)\sqrt{t}\,dt$

$$= \int (3+t)\,t^{1/2}\,dt$$
$$= \int (3t^{1/2} + t^{3/2})\,dt$$
$$= 3\int t^{1/2}\,dt + \int t^{3/2}\,dt$$
$$= \cancel{3}\cdot\frac{t^{3/2}}{\cancel{3}/2} + \frac{t^{5/2}}{5/2} + C$$
$$= \boxed{2t^{3/2} + \frac{2}{5}t^{5/2} + C}$$ D

8. $\int \frac{x^3 - 4x + 9}{x^2}\,dx$

$$= \int \left(\frac{x^{\cancel{3}\,1}}{\cancel{x^2}} - \frac{4\cancel{x}}{x^{\cancel{2}\,1}} + \frac{9}{x^2}\right)dx$$
$$= \int \left(x - \frac{4}{x} + \frac{9}{x^2}\right)dx$$
$$= \int x\,dx - 4\int \frac{1}{x}\,dx + 9\int \frac{1}{x^2}\,dx$$
$$= \int x\,dx - 4\int x^{-1}\,dx + 9\int x^{-2}\,dx$$
$$= \frac{x^2}{2} - 4\ln|x| + \frac{9}{1}\cdot\frac{x^{-1}}{-1} + C$$
$$= \frac{x^2}{2} - 4\ln|x| - 9\cdot x^{-1} + C$$
$$= \boxed{\frac{x^2}{2} - 4\ln|x| - \frac{9}{x} + C}$$ B

9. $\int (x-4)(4x+4)\,dx$ FOIL

$$= \int (4x^2 + 4x - 16x - 16)\,dx$$
$$= \int (4x^2 - 12x - 16)\,dx$$
$$= 4\int x^2\,dx - 12\int x\,dx - 16\int dx$$
$$= \frac{4}{1}\cdot\frac{x^3}{3} - \frac{\cancel{12}^{\,6}}{1}\cdot\frac{x^2}{\cancel{2}_{\,1}} - 16x + C$$
$$= \boxed{\frac{4}{3}x^3 - 6x^2 - 16x + C}$$ C

14. The limit as x approaches 0 from the left side is 4.

The limit as x approaches 0 from the right side is -1. (A)

15.
$$f(x)=\begin{cases}-1-x, & x\le 2\\ 1-2x, & x>2\end{cases}$$

Find $\lim_{x\to 2^+} f(x)$

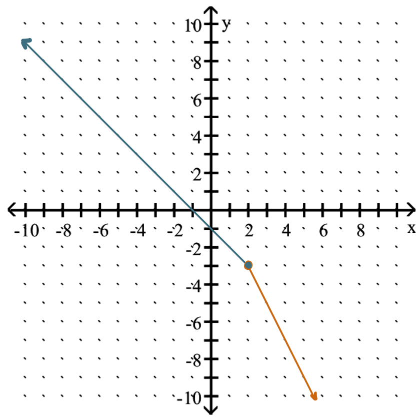
$f(x)=-1-x$

x	y
-1	0
0	-1
1	-2
2	-3

$f(x)=1-2x$

x	y
2	-3
3	-5
4	-7
5	-9

$\lim_{x\to 2^+} f(x)=-3$ (D)



16. $\lim_{x\to 2}(x^2+8x-2)$

$=(2)^2+8(2)-2$

$=4+16-2$

$=\boxed{18}$ (D)

Plug in '2'. Note that after this step, we don't write 'lim' anymore.

17. $\lim_{x\to 36}\dfrac{\sqrt{x}-6}{x-36}$

$=\lim_{x\to 36}\dfrac{\cancel{(\sqrt{x}-6)}}{\cancel{(\sqrt{x}-6)}(\sqrt{x}+6)}$

$=\lim_{x\to 36}\dfrac{1}{\sqrt{x}+6}$

$=\dfrac{1}{\sqrt{36}+6}=\dfrac{1}{6+6}=\boxed{\dfrac{1}{12}}$ (D)

We know 36 is 6^2.
Remember difference of squares:

$a^2-b^2=(a-b)(a+b)$

$(x^{1/2})^2-(6)^2=(x^{1/2}-6)(x^{1/2}+6)$

$=(\sqrt{x}-6)(\sqrt{x}+6)$

$x-36=(x^{1/2})^2-(6)^2$

18. $f(x)=10x^2$

Difference quotient:

$\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{10(x+h)^2-10x^2}{h}$

$=\dfrac{10x^2+20xh+10h^2-10x^2}{h}$

$=\dfrac{20xh+10h^2}{h}$

$=\dfrac{\cancel{h}(20x+10h)}{\cancel{h}}$

$=\boxed{(20x+10h)}$ (C)

19. $x=10$ to $x=20$

Let the point w/ $x=10$ be $A=(10,40)$, and $x=20$ be $B=(20,50)$.

If $A=(a, f(a))$, $B=(b, f(b))$, using average rate of change formula:

$$\text{Avg} = \frac{f(b)-f(a)}{b-a}$$

$$= \frac{50-40}{20-10}$$

$$= \frac{10}{10}$$

$$\text{Avg} = \boxed{1}$$

D

20. Equation of the tangent line of $f(x)=x^2-x$ @ $(\overset{x_1}{4}, \overset{y_1}{12})$

▷ Find $f'(x)$.

$f'(x) = 2x-1$

▷ Find slope $m=f'(4)$ Because $x=4$

$f'(4) = 2(4)-1$

$= 8-1$

$f'(4) = 7$ ← slope

▷ Find equation of the tangent line

$y-y_1 = m(x-x_1)$

point-slope form

$y-12 = 7(x-4)$

$y-12 = 7x-28$

$+12 \qquad +12$

$$\boxed{y=7x-16}$$

C

21. $f(x) = (4x-2)(5x^3-x^2+1)$

Product Rule Way

$f'(x) = \underset{u'}{(4)}\underset{v}{(5x^3-x^2+1)} + \underset{u}{(4x-2)}\underset{v'}{(15x^2-2x)}$

$= 20x^3-4x^2+4+60x^3-8x^2-30x^2+4x$

$$\boxed{f'(x) = 80x^3-42x^2+4x+4}$$

C

Algebra + Sum/Difference Way

Distribute:

$f(x) = (4x-2)(5x^3-x^2+1)$

$f(x) = 20x^4-4x^3+4x-10x^3+2x^2-2$

$f(x) = 20x^4-14x^3+2x^2+4x-2$

Differentiate:

$$\boxed{f'(x) = 80x^3-42x^2+4x+4}$$

22. $y = \dfrac{x}{2x-8}$ $\begin{matrix} u \\ v \end{matrix}$

$\dfrac{dy}{dx} = \dfrac{\overset{u'}{(1)}\overset{v}{(2x-8)} - \overset{u}{x}\overset{v'}{(2)}}{\underset{v^2}{(2x-8)^2}}$

$\dfrac{dy}{dx} = \dfrac{\cancel{2x}-8-\cancel{2x}}{(2x-8)^2}$

$$\boxed{\frac{dy}{dx} = -\frac{8}{(2x-8)^2}}$$

D

23. $f(x) = (2x^2+4)^5$

$f(g(x))$

$f'(x) = 5(2x^2+4)^{5-1}(4x)$

$= 5(2x^2+4)^4(4x)$

Multiply

$$\boxed{f'(x) = 20x(2x^2+4)^4}$$

B

24. $f(x)=\sqrt{1-10x}$

Rewrite ↘ $f(x)=(1-10x)^{1/2}$

$f'(x)=\frac{1}{2}(1-10x)^{1/2-1}(-10)$

$=-\frac{10}{2}(1-10x)^{1/2-2/2}$

$=-5(1-10x)^{-1/2}$

$=-\frac{5}{(1-10x)^{1/2}}$

$\boxed{f'(x)=-\frac{5}{\sqrt{1-10x}}}$

B

25. Find the second derivative.

$y=4x^4-5x^2+8$

$\frac{dy}{dx}=16x^3-10x$

$\frac{d^2y}{dx^2}=\boxed{48x^2-10}$

C

26. Find the third derivative.

$y=3x^3+5x^2-6x$

$\frac{dy}{dx}=9x^2+10x-6$

$\frac{d^2y}{dx^2}=18x+10$

$\boxed{\frac{d^3y}{dx^3}=18}$

A

27. Find the relative extrema.

$f(x)=-4x^2-2x-8$

▹ Find $f'(x)$

$f'(x)=-8x-2$

▹ Find critical value. $f'(x)=0$

$-8x-2=0$

$+2 \quad +2$

$-8x=2$

$-8 \quad -8$

$x=-\frac{1}{4}$

▹ Find critical point

(Find y) $x=-\frac{1}{4}$

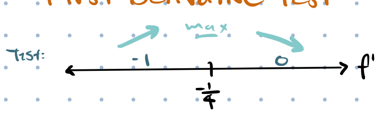
$f(-\frac{1}{4})=-4(-\frac{1}{4})^2-2(-\frac{1}{4})-8$

$=-\frac{1}{4}+\frac{1}{2}-8$

$f(-\frac{1}{4})=-\frac{31}{4}$

Gives us the point $(-\frac{1}{4},-\frac{31}{4})$

First Derivative Test



$f'(-1)=-8(-1)-2$ $\quad$ $f'(0)=-8(0)-2$

$f'(-1)=6$ ⊕ $\quad$ $f'(0)=-2$ ⊖

There is a relative max. @ $(-\frac{1}{4},-\frac{31}{4})$.

A

28. Find the relative extrema.

$f(x)=-9x^2-2x-11$

▹ Find $f'(x)$

$f'(x)=-18x-2$

▹ Find critical value. $f'(x)=0$

$-18x-2=0$

$+2 \quad +2$

$-18x=2$

$-18 \quad -18$

$x=-\frac{1}{9}$

▹ Find critical point

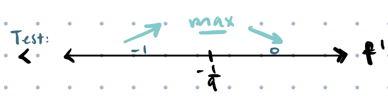
(Find y) $x=-\frac{1}{9}$

$f(-\frac{1}{9})=-9(-\frac{1}{9})^2-2(-\frac{1}{9})-11$

$f(-\frac{1}{9})=-\frac{98}{9}$

Gives us the point $(-\frac{1}{9},-\frac{98}{9})$

First Derivative Test



$f'(-1)=-18(-1)-2$ $\quad$ $f'(0)=-18(0)-2$

$f'(-1)=16$ $\quad$ $f'(0)=-2$

There is a relative max @ $(-\frac{1}{9},-\frac{98}{9})$.

A

29. $f(x) = 8x^3 + 2x + 4$

▸ Find $f''(x)$

$f'(x) = 24x^2 + 2$

$f''(x) = 48x$

▸ Find point of inflection. $f''(x) = 0$

$48x = 0$

$\frac{48x}{48} = \frac{0}{48}$

$x = 0$

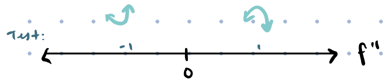
Find y:

$f(0) = 8(0)^3 + 2(0) + 4$

$y = 4$

PoI: $(0, 4)$

Second Derivative Test



$f''(-1) = -18(-1)$
$f''(-1) = 18$

$f''(1) = -18(1)$
$f''(1) = -18$

$\boxed{\text{There is a Point of inflection @ } (0, 4).}$

B

30. $s(x) = -x^2 - 24x - 135$

▸ Find $s'(x)$

$s'(x) = -2x - 24$

▸ Find critical values. $f'(x) = 0$

$-2x - 24 = 0$

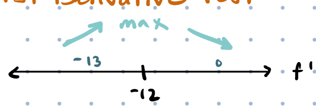
$+24 \quad +24$

$-2x = 24$

$\frac{-2x}{-2} = \frac{24}{-2}$

$x = -12$

First Derivative Test



$f'(-13) = -2(-13) - 24$
$f'(-13) = 2$

$f'(0) = -2(0) - 24$
$f'(0) = -24$

$\boxed{\text{The function is increasing on } (-\infty, -12) \text{, and decreasing on } (-12, \infty).}$

C

31. $f(x) = x^3 + 3x^2 - x - 24$

▸ Find $f''(x)$

$f'(x) = 3x^2 + 6x - 1$

$f''(x) = 6x + 6$

▸ Find point of inflection. $f''(x) = 0$

$6x + 6 = 0$

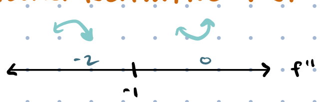
$-6 \quad -6$

$6x = -6$

$\frac{6x}{6} = \frac{-6}{6}$

$x = -1$

Second Derivative Test



$f''(-2) = 6(-2) + 6$
$f''(-2) = -6$

$f''(0) = 6(0) + 6$
$f''(0) = 6$

$\boxed{\text{The function is concave down on } (-\infty, -1) \text{, and concave up on } (-1, \infty).}$

D

32. Find the vertical asymptote.

$h(x) = \frac{3x}{x - 6}$ ← When the denominator is undefined, we obtain a vertical asymptote.

▸ Find value where $h(x)$ is undefined.

$x - 6 = 0$

$+6 \quad +6$

$x = 6$

$\boxed{\text{The function is undefined at } x = 6. \therefore \text{ the V.A. is } x = 6.}$

B

33. To find the H.A.,

$$h(x) = \frac{8x^2 - 5x - 3}{2x^2 - 6x + 2}$$

Leading coefficients have matching exponents.

Use shortcut

$$\lim_{x\to\infty} \frac{8x^2}{2x^2}$$

$$= \lim_{x\to\infty} \frac{8}{2}$$

$$= \lim_{x\to\infty} 4$$

$$= 4$$

$\boxed{\text{The horizontal asymptote is } y=4.}$

B

The long way:

$$\lim_{x\to\infty} \frac{(8x^2 - 5x - 3)\cdot\frac{1}{x^2}}{(2x^2 - 6x + 2)\cdot\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{\frac{8x^2}{x^2} - \frac{5x}{x^2} - \frac{3}{x^2}}{\frac{2x^2}{x^2} - \frac{6x}{x^2} + \frac{2}{x^2}}$$

$$= \lim_{x\to\infty} \frac{8 - \frac{5}{x} - \frac{3}{x^2}}{2 - \frac{6}{x} + \frac{2}{x^2}}$$

Apply limit

$$= \frac{8 - 0 - 0}{2 - 0 + 0}$$

$$= 4$$

★ Remember: Any number divided by a very large number (aka. ∞), equals '0'.

34. To maximize profit $P(x) = -x^3 + \frac{27}{2}x^2 - 60x + 100, \quad x \geq 5$

▹ Find $P'(x)$.

$$P'(x) = -3x^2 + \cancel{2}\cdot\frac{27}{\cancel{2}}x - 60$$

$$P'(x) = -3x^2 + 27x - 60$$

▹ Find maximum. $f'(x) = 0$

$$-3x^2 + 27x - 60 = 0$$

Factor out -3.

$$-3(x^2 - 9x + 20) = 0$$

Factor equation

$$-3(x-5)(x-4) = 0$$

$x - 5 = 0$ $x - 4 = 0$

$+5 \;\; +5$ $+4 \;\; +4$

$x = 5$ ~~$x = 4$~~

Since $x \geq 5$

4 is not less than or equal to 5.

$x = 5$ x is a hundred thousand tires

$5(100,000) = 500,000$

$\boxed{\text{500,000 tires need to be sold to maximize profit.}}$

either B or C ?

$x^2 - 9x + 20$

$a=1 \quad b=-9 \quad c=20$

$(x-5)(x-4)$

a·c: 20

-5 -4

b: -9

35. We know,

$P(x) = R(x) - C(x)$, where $R(x) = 50x - 0.5x^2$ and $C(x) = 3x + 10$.

Also,

$P'(x) = R'(x) - C'(x)$

▹ Find $R'(x)$ & $C'(x)$

$R(x) = 50x - 0.5x^2$

$R'(x) = 50 - 2(0.5)x$

$R'(x) = 50 - x$

$C(x) = 3x + 10$

$C'(x) = 3$

▹ Find $P'(x)$

$P'(x) = 50 - x - 3$

$P'(x) = 47 - x$

▹ Find max. yield profit. $P'(x) = 0$

$47 - x = 0$

$+x \quad +x$

$\boxed{x = 47 \text{ units}}$

C

36. Solve the system.

$$\begin{cases} x+y+z=0 \\ x-y+5z=-24 \\ 3x+y+z=6 \end{cases}$$

You can use a matrix.

- Create augmented matrix using coefficients.

$$\begin{array}{cccc} x & y & z & \# \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & -1 & 5 & -24 \\ 3 & 1 & 1 & 6 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -5 \end{array}\right] \quad \begin{array}{l} x=3 \\ y=2 \\ z=-5 \end{array}$$

$\boxed{\text{Solution: } (3, 2, -5)}$ A

Forgot how to row reduce?
TI-84:
[2nd] → [x^{-1}] (matrix) → MATH B:rref(

Then choose your matrix
[2nd] → [x^{-1}] (matrix) → NAMES #

37. Given $A = \begin{bmatrix} 3 & 3 \\ 2 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 6 \end{bmatrix}$

$$2A = 2\begin{bmatrix} 3 & 3 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} 2(3) & 2(3) \\ 2(2) & 2(6) \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 4 & 12 \end{bmatrix}$$

$$2A+B = \begin{bmatrix} 6 & 6 \\ 4 & 12 \end{bmatrix} + \begin{bmatrix} 0 & 4 \\ -1 & 6 \end{bmatrix} = \begin{bmatrix} 6+0 & 6+4 \\ 4-1 & 12+6 \end{bmatrix}$$

$$\boxed{2A+B = \begin{bmatrix} 6 & 10 \\ 3 & 18 \end{bmatrix}}$$ A

38. Given $A = \begin{bmatrix} -2 & 3 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 0 \\ -1 & 2 \end{bmatrix}$

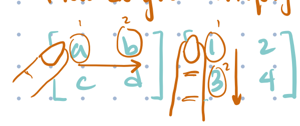
$$AB = \begin{bmatrix} -2 & 3 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} -2 & 0 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2(-2)+3(-1) & -2(0)+3(2) \\ 3(-2)+2(-1) & 3(0)+2(2) \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & 6 \\ -6-2 & 4 \end{bmatrix}$$

$$\boxed{AB = \begin{bmatrix} 1 & 6 \\ -8 & 4 \end{bmatrix}}$$ B

How do you multiply manually?



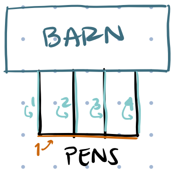
The left matrix's rows multiply the right matrix's columns.

$$= \begin{bmatrix} \underset{\text{Top row, left col.}}{a(1)+b(3)} & \underset{\text{Top row, right col.}}{a(2)+b(4)} \\ \underset{\text{Bottom row, left col.}}{c(1)+d(3)} & \underset{\text{Bottom row, right col.}}{c(2)+d(4)} \end{bmatrix}$$

or
use calculator.

[A] * [B]

39.



▷ Find constraint function.

Total fence perimeter: 160 ft.

Constraint function

Height (y), width (x)

Perimeter: $P = x + 4y$

$$160 = x + 4y$$
$${-x} \quad -x$$

$$\frac{160 - x}{4} = \frac{4y}{4}$$

$$y = \frac{160 - x}{4}$$

▷ Set up area formula. Take its derivative.

$A = h \cdot w$ (height × width)

$$A = xy$$
$$= x\left(\frac{160 - x}{4}\right)$$
$$= \frac{160x - x^2}{4}$$
$$= \frac{160}{4}x - \frac{x^2}{4}$$
$$A = 40x - \frac{1}{4}x^2$$
$$A' = 40 - \frac{1}{\cancel{4}_2} \cdot \frac{\cancel{2}^1}{1} x$$
$$A' = 40 - \frac{1}{2}x$$

▷ Make area as large as possible = Maximize area. $A' = 0$

$$40 - \frac{1}{2}x = 0$$
$$+\frac{1}{2}x \quad +\frac{1}{2}x$$
$$2(40) = \left(\frac{1}{\cancel{2}}x\right) \cdot \cancel{2}$$

x is width → $x = 80$

▷ Find height (Find y).

$$y = \frac{160 - x}{4}$$

$x = 80$

$$y = \frac{160 - 80}{4}$$

y is height →

$$y = \frac{80}{4}$$
$$y = 20$$

Width: 80 ft Height: 20 ft

B

40. We know,

$P(x) = R(x) - C(x)$, where $R(x) = 30x - 0.5x^2$ and $C(x) = 4x + 7$.

Also,

$P'(x) = R'(x) - C'(x)$

▸ Find $R'(x)$ & $C'(x)$

$R(x) = 30x - 0.5x^2$

$R'(x) = 30 - 2(0.5)x$

$R'(x) = 30 - x$

$C(x) = 4x + 7$

$C'(x) = 4$

▸ Find $P'(x)$

$P'(x) = 30 - x - 4$

$P'(x) = 26 - x$

▸ Find max. yield profit. $P'(x) = 0$

$26 - x = 0$

$+x \quad +x$

$\boxed{x = 26 \text{ units}}$

D

41. $C(x) = 120 + 3x - x^2 + 4x^3$. Find $C'(3)$.

▸ Find $C'(x)$

$C'(x) = 3 - 2x + 12x^2$

▸ Find $C'(3)$

$C'(3) = 3 - 2(3) + 12(3)^2$

$= 3 - 6 + 108$

$C'(3) = 105$

The marginal cost when $x = 3$ is \$105.

A

42. $P(x) = x^3 - 4x^2 + 8x + 5$. Find $P'(4)$

▸ Find $P'(x)$

$P'(x) = 3x^2 - 8x + 8$

▸ Find $P'(4)$

$P'(4) = 3(4)^2 - 8(4) + 8$

$P'(4) = 24$

The marginal profit when $x = 4$ is \$24.

C

43. $f(x) = \underbrace{e^{8x}}_{e^u}$

$\frac{d}{du}[e^u] = e^u \cdot du$ or $u' e^u$

$\boxed{f'(x) = \underbrace{8e^{8x}}_{u' \cdot e^u}}$

D

44. $f(x) = -6e^{5x}$

$f'(x) = -6 \cdot 5e^{5x}$

$\boxed{f'(x) = -30e^{5x}}$

B

45. $C(t) = 240 - 60e^{-t}$. Find $C'(2)$

▸ Find $C'(t)$.

$C'(t) = -60(-1)e^{-t}$

$C'(t) = 60e^{-t}$

▸ Find $C'(2)$

$C'(2) = 60e^{-(2)} \approx 8.12$

Since the cost is in millions of dollars, the marginal cost is 8.12 million dollars per year.

B

46. To find the tangent line of $f(x) = 2e^{4x}$ @ $(\overset{x}{0}, \overset{y_1}{2})$.

▸ Find $f'(x)$

$f'(x) = 2(4)e^{4x}$

$f'(x) = 8e^{4x}$

▸ Find the slope $f'(0)$ Since $x=0$

$f'(0) = 8e^{4(0)}$

$= 8e^0$

$= 8(1)$

aka slope $m = f'(0) = 8$

▸ Find the equation of the tangent line. Point: $(\overset{x}{0}, \overset{y}{2})$ Slope (m): 8

$y - y_1 = m(x - x_1)$

$y - 2 = 8(x - 0)$

$y - 2 = 8x$

$+2 \quad +2$

$\boxed{y = 8x + 2}$ (B)

47. To find the tangent line of $y = \underbrace{(x^2-x)}_{u}\underbrace{\ln(6x)}_{v}$ @ $x=2$.

▸ Find y'

$y' = \underbrace{(2x-1)}_{du}\underbrace{\ln(6x)}_{v} + \underbrace{\frac{6}{6x}}_{dv}\cdot\underbrace{(x^2-x)}_{u}$ ← Factor out an x $x^2 - x = x(x-1)$

$= (2x-1)\ln(6x) + \frac{\cancel{6x}(x-1)}{\cancel{6x}}$

$y' = (2x-1)\ln(6x) + x - 1$

▸ Plug in $x=2$ to find slope.

$m = [2(2)-1]\ln[6(2)] + 2 - 1$

$m = 3\ln(12) + 1 \approx 8.455$

▸ Find y when $x=2$

$y = [(2)^2 - (2)]\ln[6(2)]$

$y = 2\ln(12) \approx 4.97$

▸ Find the equation of the tangent line. Point: $(\overset{x_1}{2}, \overset{y_1}{4.97})$ Slope (m): 8.455

$y - y_1 = m(x - x_1)$

$y - 4.97 = 8.455(x - 2)$

$y - 4.97 = 8.455x - 16.91$

$+4.97 \quad +4.97$

$\boxed{y = 8.455x - 11.94}$ (A)

48. $y = \ln(x-6)$ $\frac{d}{du}(\ln u) = \frac{du}{u}$

$\boxed{y' = \frac{1}{x-6}}$ (D)

49. $y = \ln 2x^2$

$y' = \frac{\cancel{4}^{2}\cancel{x}}{{}_{1}\cancel{2}x^{\cancel{2}}}$

$\boxed{y' = \frac{2}{x}}$ (A)

50. $y = \dfrac{\ln x}{x^5}$ $\leftarrow u$ $\leftarrow v$

Quotient rule

$$y' = \frac{\overbrace{\frac{1}{\cancel{x}}}^{du} \cdot \overbrace{x^{\cancel{5}\,4}}^{v} - \overbrace{5x^4}^{dv}\,\overbrace{\ln x}^{u}}{\underbrace{(x^5)^2}_{v^2}}$$

$$y' = \frac{x^4 - 5x^4 \ln x}{x^{10}}$$ Factor out x^4

$$y' = \frac{\cancel{x^4}(1 - 5\ln x)}{x^{\cancel{10}\,6}}$$

$$\boxed{y' = \frac{1 - 5\ln x}{x^6}}$$

B